PHYSICAL REVIEW E **78**, 016214 (2008)

Electric-field-induced wave groupings of spiral waves with oscillatory dispersion relation

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(Received 8 May 2008; published 30 July 2008)

The dynamic behavior of spiral-shaped excitation patterns with oscillatory dispersion is investigated under the influence of externally applied direct current or alternating current. For these two types of electric field, wave-grouping phenomena are generally observed. For the direct current field, the spiral wave drifts approximately along a straight line and wave groupings appear in certain ranges of spatial polar angles when the strength of the external field is larger than a threshold. In terms of the Doppler effect induced by the drift of the spiral tip and the oscillatory dispersion, we propose a theory model to predict the spatial distribution of wave grouping and the critical strength of the current. In contrast, for the alternating current field, the spiral wave may stay stationary and wave grouping may appear in the whole space with a different manner. This finding indicates that movement of the spiral tip is not necessary for the appearance of wave grouping.

DOI: 10.1103/PhysRevE.78.016214 PACS number(s): 05.45.−a, 82.40.Ck, 47.54.−r, 82.20.Wt

I. INTRODUCTION

Spiral waves are spontaneously formed and play an important role in two-dimensional excitable or oscillatory systems, such as the classical Balousov-Zhabotinsky (BZ) reaction [1], cardiac tissues [2], Dictyostelium discoideum amoebae population [3], and surface chemical reactions [4,5]. Studies of spiral waves have continuously attracted the interest of researchers for several decades due to the significance of both theoretic development and potential applications in pattern formation. The simplest form of a spiral is a rigidly rotating spiral; namely, the spiral sends out a traveling wave and its tip moves periodically along a round circle. As a certain system parameter is varied, this spiral may become unstable through a Hopf bifurcation and a meandering spiral appears with the tip trajectory showing cycloid [6–8]. With the system parameter being changed further, the meandering spiral may also get unstable and break up and a turbulent pattern with defects is generally observed.

The spiral wave dynamics is determined by the properties of the medium such as the dispersion relation of the medium, which shows the dependence of the speed, $V(\lambda)$, of a periodic wave train on the wavelength λ. Generally speaking, the longer the interval between pulses (period or wavelength), the more thoroughly the medium has recovered, the lower the threshold for next excitation, and the greater the speed of a propagating wave front is. Thus, the velocity of wave trains, $V(\lambda)$, is a monotonically increasing function of λ; this characteristic shows a strict restriction for a stable wave train. Usually for the normal dispersion relation [9–13], $V(\lambda)$ monotonically increases with λ and converges to the velocity of a solitary pulse for large wavelength, and the wavelength has a minimum value below which no wave trains exist. However, researchers have also observed some other types of dispersion relation deviating from the normal one—for example, curves with damped oscillations (oscillatory dispersion) [14], a single overshoot [15], bistability [16], and band gaps [17]. Such anomalous dispersions give rise to some exotic phenomena that cannot be found in media with normal dispersion. For instance, wave bunching was observed in a one-dimensional FitzHugh-Nagumo (FHN) medium [18]. In a modified BZ reaction using 1,4-cyclohexanedione as the organic substrate, Manz *et al.* found bunching, stacking, and merging of waves [19]. Later on with the same chemical equipment they observed phenomena of propagation failure, breathing pulses, and backfiring pulse [15].

In a recent BZ experiment, a new type of meandering spiral pattern, a wave-grouped spiral [20], was observed in excitable media with oscillatory dispersion, in which the dense waves form groups while the sparse waves remain evenly spaced. The underlying mechanism for wave grouping has been well addressed, and it is due to the coaction of the Doppler effect of the meandering spiral and the oscillatory dispersion relation of the system. Very recently, the same research group considered the effect of a parameter gradient in a three-dimensional system on wave grouping [21]. The experimental observation has been well supplemented by numerical study. In simulations, the central tip was artificially dragged to imitate the meandering spiral in the experimental condition. It seems necessary to do so, as the Doppler effect induced by the moving of spiral tip is believed to be one of the key ingredients for wave grouping. These studies prompt some interesting questions: Can we find wave grouping in a more natural way? Is the wave grouping phenomenon generic? And what is the necessary condition for wave grouping?

In this paper, we attempt to answer these questions by studying the dynamical behaviors of spirals with oscillatory dispersion in the presence of an external field (direct or alternating electric field). Electric fields are well known to play important roles in the behaviors of spiral waves. For example, the drift of spiral waves was observed in the presence of a constant direct current [22–25] and a resonance drift of spiral waves subject to electric pulses of alternating polarity (a square-wave-form alternating current) was observed [26]. A recent work showed that an alternating electric field can even be used to eliminate spiral breakup [27]. Most of previous studies focused only on the dynamics or control

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method for generating a drift by imposing an external field on the spiral with normal dispersion, and much less was known for that with anomalous dispersion [28]. An intuitive idea is that the external field can naturally produce a drift of the spiral, which gives rise to a Doppler effect in the domain, and thus make wave grouping observable. In this work, for the direct current, a wave grouping similar to the meandering-spiral-induced wave grouping in [20] is observed. A model is developed to explain these phenomena. Under the condition of an alternating current field, surprisingly we find that the spiral tip may not drift and the spirals may form a group in a novel manner. In this case, it is a pure effect of oscillatory dispersion and obviously the Doppler effect is not involved.

This paper is organized in the following order. In Sec. II, we present our observations of wave grouping with a direct current field and give a model to explain these observations. In Sec. III, we study the corresponding dynamics with an alternating sinusoidal current field. Section IV is devoted to a brief summary.

II. WAVE GROUPING OF SPIRAL WAVES IN A DIRECT CURRENT ELECTRICAL FIELD

We consider the FHN model

$$\frac{\partial A}{\partial t} = \frac{1}{\epsilon}(A - A^3/3 - B) + D\nabla^2 A,$$

$$\frac{\partial B}{\partial t} = \epsilon(A + \beta - \gamma B), \tag{1}$$

where $A(x,y,t)$ and $B(x,y,t)$ are the fast and slow variables, respectively, and x, y, and t denote the spatial coordinates and time. D is the diffusion coefficient of the fast variable. For the local dynamics, whenever $0<\gamma<1$, $\gamma\epsilon^2<1$, $|\beta| = \beta_H = (1-\gamma\epsilon^2)^{1/2}[(2\gamma+\gamma^2\epsilon^2)/3-1]$, where β_H denotes a Hopf bifurcation parameter. For $|\beta|<\beta_H$, the FHN system exhibits oscillations; otherwise, for $|\beta|>\beta_H$, it is excitable. In this paper, the parameters are chosen to be the same as those in the literature to generate damped oscillations in the dispersion curve [7,12], as illustrated in Fig. 1. The parameters $\epsilon=0.3$, $\gamma=0.5$, $\beta=0.7$, and $D=1.0$ are unchanged throughout the paper. Under such parameters $\epsilon=0.3$ and $\gamma=0.5$, $\beta_H=0.644$. Here $\beta=0.7\gtrsim\beta_H$, as to obtain an oscillatory dispersion relation, the parameter set has to be chosen to be very close to Hopf bifurcation and within the excitable regime [7]. In this situation, the free stable spiral wave is a rigid rotation one with the period $T_0\approx 11.22$ and the wavelength $\lambda_0\approx 22.0$, as shown by the arrow and the first open circle in Fig. 1. The wave velocity $V_0=\lambda_0/T_0\approx 1.96$. The maximum $V(\lambda)$ is located at $\lambda_c\approx 25.75$, the end point of the first ascending branch. Below we will see that λ_c plays a significant role in the establishment of wave grouping.

When applying an advective electric field parallel to the y axis, we have

$$\frac{\partial A}{\partial t} = \frac{1}{\epsilon}(A - A^3/3 - B) + D\nabla^2 A + E\frac{\partial A}{\partial y},$$

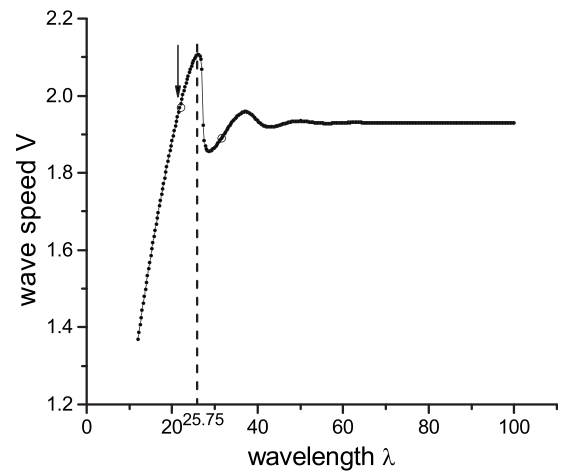

FIG. 1. An oscillatory dispersion curve of the FHN model shown as the wave speed V vs the wavelength λ. $\epsilon=0.3$, $\gamma=0.5$, $\beta=0.7$, and $D=1.0$. At $\lambda_c=25.75$, V is largest. The first open circle pointed out by the vertical arrow at $\lambda_0=22.0$ denotes the location of the free spiral, while the second open circle at $\lambda=31.5$ denotes the location of the newly formed stable spiral under the alternating current field $(1.05<E<1.19)$.

$$\frac{\partial B}{\partial t} = \epsilon(A + \beta - \gamma B) + \delta E\frac{\partial B}{\partial y}. \tag{2}$$

Here E describes the strength of the electric field and $\delta \in [0,1]$ is the ratio of the effects of the electric field on the two variables. No-flux boundary conditions are considered. We numerically solved this model with the simple explicit Euler scheme: We used a time step $\Delta t=0.01$ and a space step $h=0.5$, as well as a square spatial grid of size 1280×1280 nodes.

In the following, we pay attention to the responses of spirals to such an external field and focus on wave-grouping structure. Usually $E\in[0,1.0]$ is tuned with a fixed δ ($\delta=0.3$). In our simulations, very rich spiral structures with variance of E are observed: When E is lower than 0.24, the spiral is stable with a small drifting speed. However, if E is increased beyond 0.24, wave-grouped structures begin to appear with several wave fronts grouped together in a certain spatial polar angle, while the wave fronts in the other spatial domain are still evenly spaced, and the drift velocity becomes larger. Roughly, the range of wave grouping expands with the increase of E. For $0.24<E<0.5$, we observe two-wave grouping, 3-wave grouping, and combinative 2- and 3-wave grouping. These wave-grouped structures are independent of initial conditions and remain stable for a fairly long time. As an example, see Fig. 2(a) for $E=0.4$. Some more complicated 4-, 5-, and even 7-wave groupings can be found in transient processing. For $0.5<E<0.8$, only 2-wave grouping is observed, but the pattern is divided into four districts with two uniform waves and two 2-wave groupings [Fig. 2(b)]. For $0.8<E<0.95$, as shown in Fig. 2(c), the wavelengths of the sparse waves become so large that it is difficult to decide whether the waves in some parts of domain are uniform or not. Finally, at $E=0.95$, excitation waves of dense waves become too thinner to be sustained

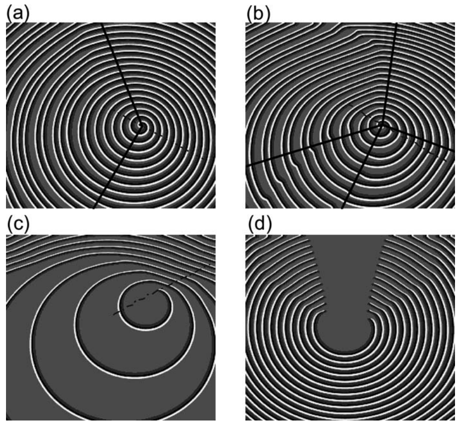

FIG. 2. Snapshots of spiral patterns in direct current electric field of different strengths: (a) $E=0.4$. Two heavy lines give the range of wave grouping: $1.91<\theta<4.23$. (b) $E=0.7$. Four heavy lines give the ranges of two 2-wave groupings: $1.45<\theta<3.47$ and $4.30<\theta<5.89$. (c) $E=0.8$. (d) $E=1.0$. In (a)–(c), the tip trajectories are indicated by dashed lines; all spirals drift from center to boundary. $\delta=0.3$.

and are split [Fig. 2(d)]; this finding is similar to the crescent-shaped waves found in Ref. [22].

In addition to these phenomenal observations, we numerically calculated the speed and direction for the spiral drift. The results are shown in Figs. 3(a) and 3(b). Roughly the tip drifts to the lower right region and the direction only slightly changes with E. The speed increases with E with a linear part discernible for small E. These findings are consistent with previous observations for spiral drift under direct current field with a normal dispersion relation [11].

As now the wave grouping is limited in a certain spatial range, it should be valuable to analyze its structure and understand its mechanism. The Doppler effect shows that when a wave source moves, the effective wavelengths will be shortened (compressed waves) in front of it and become longer (dilated waves) in the opposite direction, and the period (or frequency) varies accordingly,

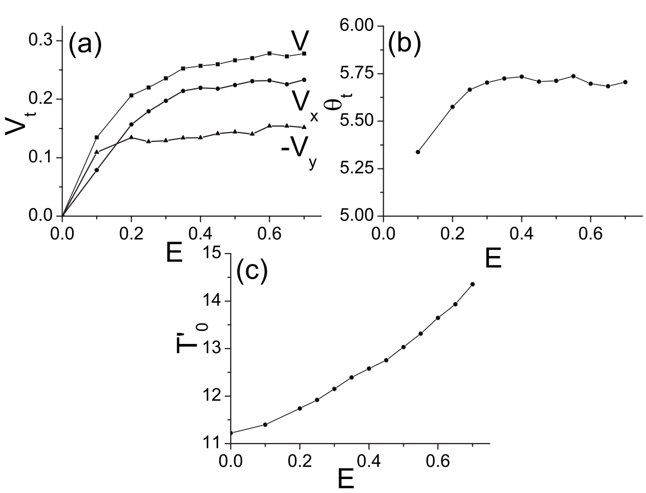

FIG. 3. (a), (b), and (c) Plots of V_t, θ_t, and T_0' versus E, respectively. $\delta=0.3$.

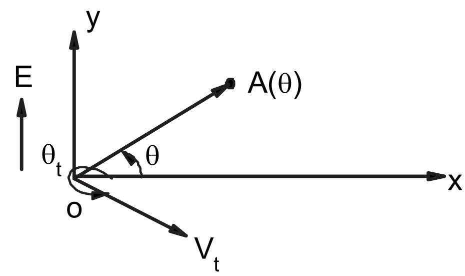

FIG. 4. Schematic show for the analysis of wave grouping induced by tip drift under the direct current electric field.

$$T' = \left(1 \mp \frac{V_s}{V}\right) T, \tag{3}$$

with the wave velocity unchanged, where V and V_s denote the speed of the wave and the wave source (equivalently, the drifting speed of the spiral tip in the present situation), respectively, and T and T' indicate the wave period and the local period after the wave drifts. "−" describes compressed waves in front of the wave drift, and "+" is for dilated waves behind the wave drift.

For our specific problem, we have to extend the above equation (for the directions of wave and wave source being the same or opposite in one line) to the analysis of whole space. We give a schematic of our analysis in Fig. 4, where V_t and θ_t denote the velocity and polar angle of the tip drift, and need to obtain the information of local period (or frequency) and wavelength at any arbitrarily chosen spatial point $A(\theta)$. In Eqs. (2), the external field on the fast variable can be divided into two parts δE and $(1-\delta)E$. The δE part on the fast variable and that on the slow variable simply result in a pure drift of the whole pattern (not the tip drift) with the speed δE antiparallel to the direction of electric field (i.e., $-y$ direction in our study), as we can easily eliminate these two advective δE terms in Eqs. (2) by changing the system to a comoving frame $y_1=y+\delta Et$. In this way, we have to eliminate this common part in the wave velocity V_0. Note that V_0 is the velocity of spiral wave fronts without tip drifting ($E=0$). After considering this point and the projection of vectors on the polar angle θ and assuming the direction of V_0 being at θ (actually V_0 is perpendicular to the wave front of spiral), we have $V=V_0-\delta E\sin\theta$ and $V_s=V_t\cos(\theta_t-\theta)$. Thus, we obtain

$$T'(\theta) = \left(1 - \frac{V_t\cos(\theta_t-\theta)}{V_0-\delta E\sin\theta}\right) T_0'. \tag{4}$$

Similarly, we get the formula for the local wavelength $\lambda'(\theta)$,

$$\begin{aligned}\lambda'(\theta) &= T'(V_0-\delta E\sin\theta)\\ &= [V_0-\delta E\sin\theta - V_t\cos(\theta_t-\theta)]T_0'. \end{aligned}\tag{5}$$

The wave period T_0' is constant and is a function of E. $T_0'(E=0)=T_0$. Generally, we cannot get $T_0'(E)$ by analysis, but numerically it is easy. Usually we make the pattern evolve a short period, get a time series of one reference point far from the core and within the non-wave-grouping regime [for example, the white dot in Fig. 5(a) and its corresponding

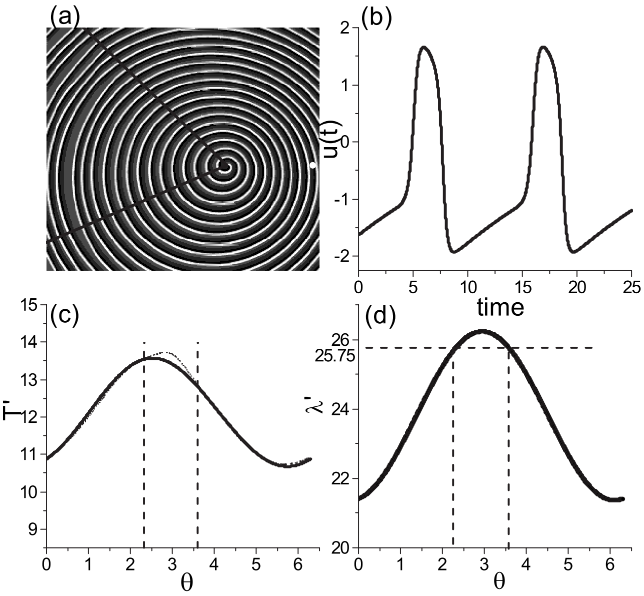

FIG. 5. Analysis of the spatial range of wave grouping. (a) Two heavy radial lines from the analysis of (c) and (d) give the range of wave grouping: $2.30 < \theta < 3.59$. (b) Time series of one reference point [the white circle chosen from (a)] with a local period $T'(\theta=0)=10.93$. Thus, we obtain $T'_0=12.15$. (c) and (d) T' and λ' as a function of θ, respectively. In (c), the solid line comes from our prediction, whereas the dotted line is from measurement. Obviously we can find the region of wave grouping from the deviation part in (c) and $\lambda' > \lambda_c = 25.75$ part in (d). $E=0.3$ and $\delta=0.3$.

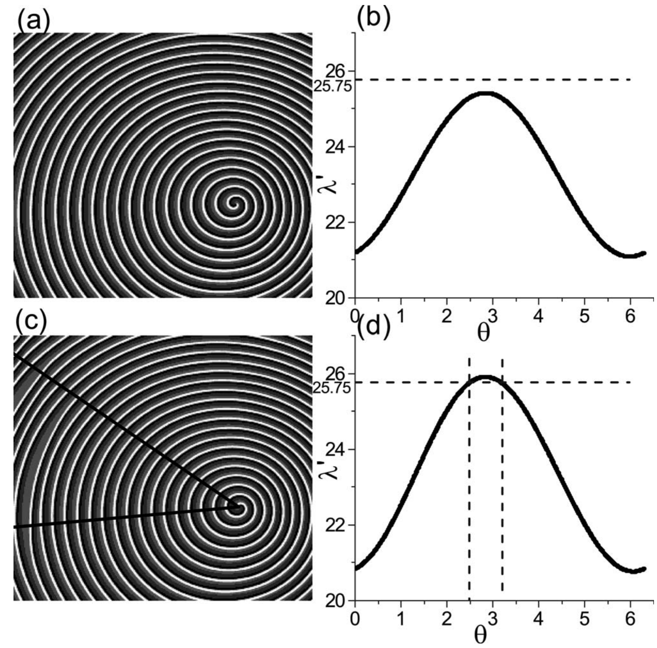

FIG. 6. Snapshots of spiral pattern in the direct current electric field in the absence of wave grouping [$E=0.23$ (a)] and in the presence of wave grouping [$E=0.25$ (c)], and their corresponding λ' distributions in (b) and (d), respectively. The two vertical dashed lines in (d) for $\lambda' > \lambda_c = 25.75$ determine the range of wave grouping [$2.49 < \theta < 3.20$ in (c)]. The wave grouping happens once the $\lambda'(\theta)$ curve touches the threshold $\lambda_c=25.75$, which signals that the critical value for the appearance of wave grouping is $E=0.24$.

time series in Fig. 5(b)], determine its local period T', and obtain T'_0 further from Eq. (4). The dependence of T'_0 on E is shown in Fig. 3(c).

Below we can easily predict the spatial distribution of wave grouping. Usually, for the wave fronts staying at the first ascending branch of the oscillatory curve, they are stable and wave grouping is impossible. However, if some wave fronts cross λ_c and move into the descending branch, they become unstable and wave grouping occurs. Take the pattern in Fig. 5(a), for example. Here $E=0.3$ and $\delta=0.3$, and $V_t=0.236$, $\theta_t=5.703$, and $T'_0 \approx 12.15$, whose value is obtained from the analysis of one single point. From Eqs. (4) and (5), we get $T'(\theta)$ and $\lambda'(\theta)$ and plot them as a function of θ in Figs. 5(c) and 5(d), respectively. These theoretic predictions are indicated by solid curves and the value of local period from measurement is plotted with a dashed curve in Fig. 5(c) for comparison. A distortion is clear within $2.32 < \theta < 3.59$, and at the same region, $\lambda' > \lambda_c = 25.75$, as shown in Fig. 5(d). As a result, the wave-grouping region in Fig. 5(a) can be predicted by the two heavy radial lines, and this prediction is in good agreement with the real pattern. For the other two cases in Figs. 2(a) and 2(b), the predictions are also good.

With the same theory, we can even predict the critical strength of the external field for wave grouping. The two snapshots of spiral pattern without and with wave grouping are presented in Figs. 6(a) and 6(c) for $E=0.23$ and 0.25, respectively. Figures 6(b) and 6(d) plot their corresponding λ' distribution. For $E=0.23$, all $\lambda'(\theta)$'s are smaller than $\lambda_c=25.75$, which implies that all wave fronts stay in the first ascending branch of the dispersion curve and are stable. For $E=0.25$, oppositely, there is a small region for $\lambda' > \lambda_c$, which indicates the spatial position for wave grouping. Again it fits well [Fig. 6(c)]. Based on these comparisons, the threshold for the appearance of wave grouping ($E=0.24$) is obtained.

III. WAVE GROUPING OF SPIRAL WAVES IN AN ALTERNATING CURRENT ELECTRIC FIELD

So far, we have investigated wave grouping in a direct current electric field. Below we will study the dynamics in an alternating current electric field. The reaction-diffusion equation becomes

$$\frac{\partial A}{\partial t} = \frac{1}{\epsilon}(A - A^3/3 - B) + D\nabla^2 A + E\cos\left(\frac{2\pi t}{T}\right)\frac{\partial A}{\partial y},$$

$$\frac{\partial B}{\partial t} = \epsilon(A + \beta - \gamma B), \tag{6}$$

where E and T are the strength and period of the external field, respectively.

Without losing generality, E is again slowly tuned with a fixed T ($T=T_0/2=5.61$). Figures 7(a)–7(f) show the snapshots of spiral-shaped patterns for $E=0.1$, 0.3, 0.5, 0.7, 1.1, and 1.25, respectively. For $0 < E < 0.22$, the spiral is stable and λ is unchanged with E. Here $\lambda=\lambda_0=22.0$, as shown by the first open circle in the dispersion curve in Fig. 1. For

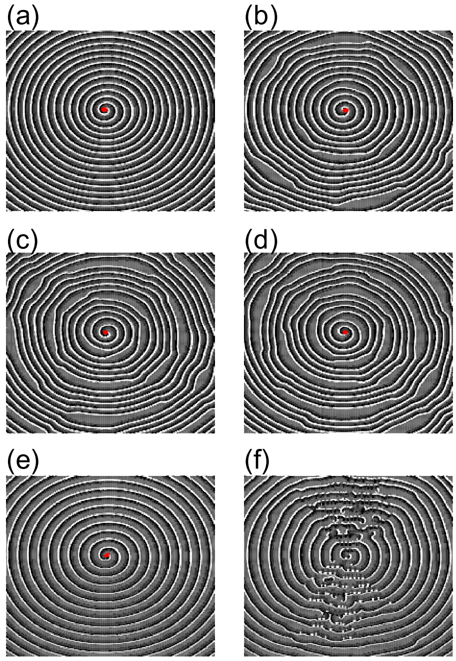

FIG. 7. (Color online) (a)–(f) Snapshots of spiral pattern in alternating current electric field for different intensities: $E=0.1$, 0.3, 0.5, 0.7, 1.1, and 1.25, respectively. $T=T_0/2=5.61$. The tip trajectories are superimposed.

$0.22<E<1.04$, very rich wave grouping phenomena can be found. Now waves become irregular, wiggling, and separated by some empty spaces (unexcited regions). They form groups in any position of the space (not specific region). Roughly, these empty areas get wider with an increase of E [comparing Figs. 7(b)–7(d)]. In a very long observation time, the tip seems stationary and no drift is found. However, we really observed an extremely small modulated motion of the spiral tip and the change from a rigidly rotating spiral [Fig. 7(a)] to a meandering one [Fig. 7(b)]. The tip trajectories are superimposed in the patterns in Fig. 7 and the very small modulated motions of tips restricted in a small region in Figs. 7(b)–7(d) is nearly unrecognizable. We also observed extremely slow movement and rearrangement of these wiggling spirals with time (for example, see Fig. 8 for the pattern evolution, $E=0.7$). All these observations well demonstrate that this is a new type of wave grouping, which is obviously different from wave grouping in direct current field or the wave grouping induced by meandering spiral in the experiment. A key distinguishing feature of this novel pattern is that the tip is nearly fixed. Therefore, the Doppler effect, which is necessary for the usual wave grouping, is not required now. For $1.04<E<1.2$, a rigidly rotating spiral occurs again, as shown in Fig. 7(e). Now λ becomes larger ($\lambda\approx 3\lambda_0/2=31.5$), which is shown in Fig. 1 with the second open circle. Clearly, the spiral has jumped from the first ascending branch to the second one in the dispersion curve. For E increasing further ($E>1.2$), the breakup of this regular spiral is observed, as shown in Fig. 7(f) for $E=1.25$.

Some other values of T were also studied and some qualitatively similar observations for non-tip-drift wave grouping were found. It is easy to understand that as the electric pulses of alternating polarity and intensity are periodically changed in such an advective field, the overall effect is a stationary (or approximately stationary) spiral. However, we really observed resonance drifts of spiral for some specifically chosen T's similar to the usual observations for normal dispersion [26]. Both sinusoidal and square-wave-form alternating current fields were studied. If the resonance drift of spirals happens, the usual tip-drift-induced wave grouping with the pattern similar to that under the direct current field condition can be found again.

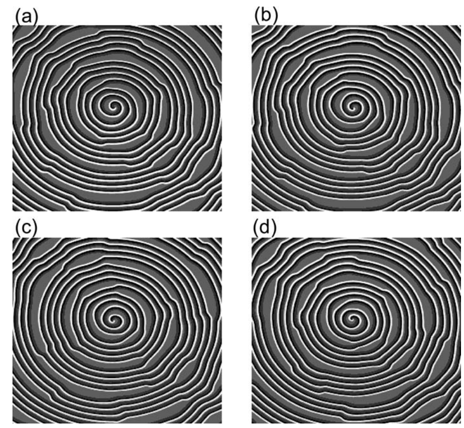

FIG. 8. (a)–(d) Evolution of wave-grouped spirals in alternating current electric field at $t=T_s$, $2T_s$, $10T_s$, and $100T_s$, respectively. T_s ($T_s=12.0$) is the rotation period of the spiral tip. A long transient has been discarded. $E=0.7$ and $T=T_0/2=5.61$.

IV. SUMMARY

In summary, we have studied the dynamical behaviors of spiral waves in excitable media with oscillatory dispersion for the direct current field and alternating current field and we have found two different types of wave grouping for these two fields. First, both wave grouping phenomena are determined by the main characteristic of the medium: the oscillatory dispersion relation. In the direct current field, the spiral drifts and wave grouping appear in a certain spatial region, as the wave fronts within such a region are unstable due to the Doppler effect, which is produced by the spiral drift, and they have to rearrange with a new wave form (wave grouping). In the alternating current field, the spiral may not drift and waves may form groups randomly in the whole domain. Here the Doppler effect does not exist, but the wave fronts become unstable due to the input of the external field and have to rearrange their wavelengths in a more uniform way. For sufficiently large driving intensity, an evenly spaced spiral with a larger wavelength located at the second ascending branch of the oscillatory dispersion curve can be observed. However, the underlying mechanisms for the stable-unstable transition of waves due to the effect of the oscillatory dispersion relation are the same. In these two cases, for the appearance of wave grouping, a sufficiently large strength of the external field is needed. Second, compared with the spirals with normal dispersion in the presence of external electric fields, some features are the same in the present study with anomalous dispersion, such as the drift in the direct current field and the resonance drifts for certain driving frequencies in the alternating current field. The chirality effect with oscillatory dispersion in the direct current field was also investigated and a phenomenon similar to the experimental observation with normal dispersion (namely, the component of the drift velocity perpendicular to the field changes its sign with the chirality of spiral) was found [11]. Nevertheless, the wave grouping is definitely unique for anomalous dispersion only. Third, from our observations, we know that the Doppler effect (or the spiral drift) actually is unnecessary in some circumstance. This finding is clearly distinct from what was expected previously [20,21,28]. Finally, we expect that our numerical observations and theoretic results in the paper can be justified in experiments.

ACKNOWLEDGMENTS

This work was partially supported by the Outstanding Oversea Scholar Foundation of Chinese Academy of Sciences and the National Natural Science Foundation of China under Grant No. 10675161.

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